

Mathematics question paper with solutions

Drop.

25.11.2.17

1. The values of  $\theta \in (0, 2\pi)$  for which  $2 \sin^2\theta - 5 \sin\theta + 2 > 0$ , are

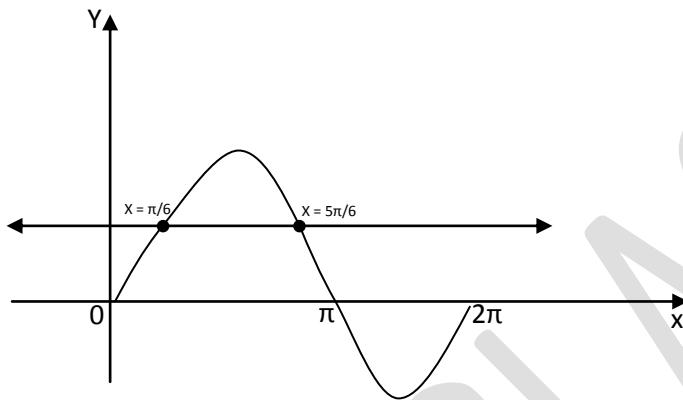
[2006-3M, -1]

- (a)  $(0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$       (b)  $(\frac{\pi}{8}, \frac{5\pi}{6})$       (c)  $(0, \frac{\pi}{8}) \cup (\frac{\pi}{6}, \frac{5\pi}{6})$       (d)  $(\frac{41\pi}{48}, \pi)$

Ans. a,  $2 \sin^2\theta - 5 \sin\theta + 2 > 0$

$$\Rightarrow (\sin\theta - 2)(2 \sin\theta - 1) > 0$$

$$\Rightarrow \sin\theta < \frac{1}{2} \quad [\because -1 \leq \sin\theta \leq 1]$$



From graph, we get  $x \in (0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$

2. A man walks a distance of 3 units from the origin towards the north-east ( $N 45^\circ E$ ) direction. From there, he walks a distance of 4 units towards the north-west ( $N 45^\circ W$ ) direction to reach a point P. then the position of P in the Argand plane is

[2007-3M]

- (a)  $3e^{i\pi/4} + 4i$       (b)  $(3 - 4i)e^{i\pi/4}$       (c)  $(4 + 3i)e^{i\pi/4}$       (d)  $(3 + 4i)e^{i\pi/4}$

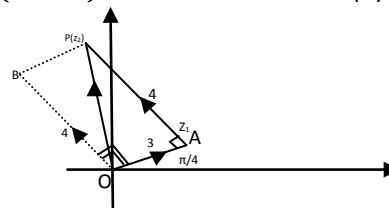
Ans. d,  $\vec{OP} = \vec{OA} + \vec{AP}$

$$\Rightarrow \vec{OP} = \vec{OA} + \vec{OB}$$

$$\Rightarrow \vec{OP} = 3e^{i\pi/4} + 4e^{i(\pi/2 + \pi/4)}$$

$$= 3e^{i\pi/4} + 4e^{i\pi/4} + 4e^{i(\pi/2 + \pi/4)}$$

$$= 3e^{i\pi/4} + 4ie^{i\pi/4} = e^{i\pi/4} (3 + 4i).$$



3. Let  $z = x + iy$  be complex number where  $x$  and  $y$  are integers. Then the area of the rectangle whose vertices are the roots of the equation  $z \bar{z}^3 + \bar{z} z^3 = 350$  is

[2009]

- (a) 48      (b) 32      (c) 40      (d) 80

Ans. a, Given  $z = x + iy$  where  $x$  and  $y$  are integer

$$\text{Also } z\bar{z}^3 + \bar{z}z^3 = 350 \Rightarrow |z|^2 (\bar{z}^2 + z^2) = 350$$

$$\Rightarrow (x^2 + y^2)(x^2 - y^2) = 175$$

$$\Rightarrow (x^2 + y^2)(x^2 - y^2) = 25 \times 7 \quad \dots(i)$$

$$\text{or } (x^2 + y^2)(x^2 - y^2) = 35 \times 5 \quad \dots(ii)$$

$\because$   $x$  and  $y$  are integers,

$$\therefore x^2 + y^2 = 25 \text{ and } x^2 - y^2 = 7 \quad [\text{From eq (i)}]$$

$$\Rightarrow x^2 = 16 \text{ and } y^2 = 9$$

$$\Rightarrow x = \pm 4 \text{ and } y = \pm 3$$

$\therefore$  Vertices of rectangle are

$$(4,3), (4, -3), (-4, -3), (-4,3).$$

So, area of rectangle =  $8 \times 6 = 48$  sq. units

Now form eq.(ii)

$$\text{Or } x^2 + y^2 = 35 \text{ and } x^2 - y^2 = 5$$

$$\Rightarrow x^2 = 20, \text{ which is not possible for any integral value of } x$$

4. The set of all real numbers  $x$  for which  $x^2 - |x + 2| + x > 0$ , is

(a)  $(-\infty, -2) \cup (2, \infty)$

(b)  $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

(c)  $(-\infty, -1) \cup (1, \infty)$

(d)  $(\sqrt{2}, \infty)$

**Ans. b,** For  $x < -2$ ,  $|x + 2| = x + 2$  and the inequality becomes

$$x^2 + x + 2 + x > 0 \Rightarrow (x + 1)^2 + 1 > 0$$

Which is valid  $\forall x \in \mathbb{R}$  but  $x < -2$

$$\therefore x \in (-\infty, -2) \quad \dots(1)$$

For  $x \geq 2$ ,  $|x + 2| = x + 2$  and the inequality becomes

$$x^2 - x - 2 + x > 0 \Rightarrow x^2 > 2 \Rightarrow x > \sqrt{2} \text{ or } x < -\sqrt{2}$$

$$\text{i.e., } x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$$

$$\text{but } x \geq -2 \Rightarrow x \in [-2, -\sqrt{2}) \cup (\sqrt{2}, \infty) \quad \dots(2)$$

From (1) and (2)

$$x \in (-\infty, -2) \cup [-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$$

$$\Rightarrow x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$$

5. The quadratic equation  $p(x) = 0$  with real coefficients has purely imaginary roots. Then the equation

$p(p(x)) = 0$  has

[JEE Adv. 2014]

(a) One purely imaginary root

(b) all real roots

(c) Two real and two purely imaginary roots

(d) **neither real nor purely imaginary roots**

**Ans. d,** Quadratic equation with real coefficients and purely imaginary roots can be considered as

$$p(x) = x^2 + a = 0 \text{ where } a > 0 \text{ and } a \in \mathbb{R}$$

$$\text{The } p[p(x)] = 0 \Rightarrow (x^2 + a)^2 + a = 0$$

$$\Rightarrow x^4 + 2ax^2 + (a^2 + a) = 0$$

$$\Rightarrow x^2 = \frac{-2a \pm \sqrt{4a^2 - 4a^2 - 4a}}{2}$$

$$\Rightarrow x^2 = -a \pm \sqrt{a}i$$

$$\Rightarrow x = \sqrt{-a \pm \sqrt{a}i} = a \pm i\beta \text{ where } a, \beta \neq 0$$

$\therefore p[p(x)] = 0$  has complex roots which are neither purely real nor purely imaginary.

6. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each

envelope contains exactly one card and no card is placed in the envelope bearing the same number and

moreover the card number 1 is always placed in envelope numbered 2. Then the number of ways it can be

done is

[JEE Adv. 2014]

(a) 264

(b) 265

(c) **53**

(d) 67

**Ans. c,**  $\therefore$  Card numbered 2 is placed in envelope numbered 2, we can consider two cases.

**Case I:** Card numbered 2 is placed in envelope numbered 1. Then it is dearrangement of 4 objects, which

$$\text{can be done in } 4! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = 9 \text{ ways}$$

**Case II:** Card numbered 2 is not placed in envelope numbered 1.

Then it is dearrangement of 5 objects, which can be done in  $5! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 44$  ways

$\therefore$  Total ways =  $44 + 9 = 53$

7. How many different nine digit numbers can be formed from the number 23355888 by rearranging its digits so that the odd digits occupy even positions? [2000S]

(a) 16 (b) 36 (c) 60 (d) 180

**Ans. c.**  $X - X - X - X - X$ . The four digits 3,3,5,5 can be arranged at (-) Places in  $\frac{4!}{2!2!} = 6$  ways.

The five digits 2,2,8,8,8 can be arranged at

(X) places in  $\frac{5!}{2!3!} = 10$  ways.

Total no. of arrangements =  $6 \times 10 = 60$  ways.

8. The sum of the first  $n$  terms of the series  $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \dots$  is  $\frac{n(n+1)^2}{2}$  when  $n$  is even. When  $n$  is odd the sum is [2004]

(a)  $\left[\frac{n(n+1)}{2}\right]^2$  (b)  $\frac{n^2(n+1)}{2}$  (c)  $\frac{n(n+1)^2}{4}$  (d)  $\frac{3n(n+1)}{2}$

**Ans. b.** If  $n$  is odd, the required sum is

$$1^2 + 2.2^2 + 3^2 + 2.4^2 + \dots + 2.(n-1)^2$$

$$= \frac{(n-1)(n-1+1)^2}{2} + n^2$$

[ $\because (n-1)$  is even  $\therefore$  using given formula for the sum of  $(n-1)$  terms.]

$$= \left(\frac{n-1}{2} + 1\right)n^2 = \frac{n^2(n+1)}{2}$$

9. If  $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$  where  $a, b, c$  are in A.P and  $|a| < 1, |b| < 1, |c| < 1$  then  $x, y, z$  are in [2005]

(a) G.P. (b) A.P.  
(c) Arithmetic- Geometric Progression (d) H.P.

**Ans. d.**  $x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$       $a = 1 - \frac{1}{x}$   
 $y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b}$       $b = 1 - \frac{1}{y}$   
 $z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c}$       $c = 1 - \frac{1}{z}$

$a, b, c$  are in A.P. or  $2b = a + c$

$$2\left(1 - \frac{1}{y}\right) = 1 - \frac{1}{x} + 1 - \frac{1}{z}$$

$$\frac{2}{y} = \frac{1}{x} + \frac{1}{z} \Rightarrow x, y, z \text{ are in H.P.}$$

10. The sum to infinite term of the series  $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots$  is [2009]

(a) 3 (b) 4 (c) 6 (d) 2

**Ans. a.** we have

$$S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots \infty \quad \dots(1)$$

Multiplying both sides by  $\frac{1}{3}$  we get

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \dots \infty \quad \dots(2)$$

subtracting eqn. (2) from eqn. (1) we get

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \dots \infty$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots \infty$$

$$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1-\frac{1}{3}} = \frac{4}{3} \times \frac{3}{2} \Rightarrow S = 3$$

11. The points  $(-a, -b)$ ,  $(0, 0)$ ,  $(a, b)$  and  $(a^2, ab)$  are :

- (a) **Collinear** (b) vertices of a parallelogram  
(c) Vertices of a rectangle (d) None of these

**Ans. a**, the given points are  $A(-a, -b)$ ,  $B(0,0)$ ,  $C(a, b)$  and  $D(a^2, ab)$ .

$$\text{Slope of } AB = \frac{b}{a} = \text{slope of } BC = \text{slope of } BD$$

$\therefore$  A, B, C, D are collinear.

12. The straight lines  $x + y = 0$ ,  $3x + y - 4 = 0$ ,  $x + 3y - 4 = 0$  form a triangle which is

[1983 -1M]

- (a) **Isosceles** (b) equilateral (c) right angled (d) none of these

**Ans. a**, solving the given equations of lines pairwise, we get the vertices of  $\Delta$  as

$$A(-2,2)B(2,-2)C(1,1)$$

$$\text{Then } AB = \sqrt{16 + 16} = 4\sqrt{2}$$

$$BC = \sqrt{1 + 9} = \sqrt{10}$$

$$CA = \sqrt{9 + 1} = \sqrt{10} \quad \therefore \Delta \text{ is isosceles.}$$

13. If  $x$  satisfies  $|x - 1| + |x - 2| + |x - 3| \geq 6$ , then,

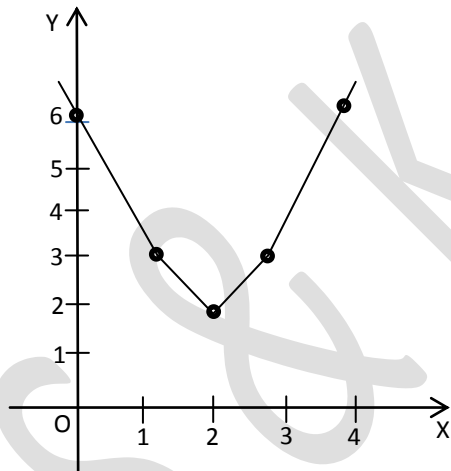
- (a) (b)  $x \leq -2$  or  $x \geq 4$  (c)  $x \leq 0$  or  $x \geq 4$  (d) none of these

**Ans. c**,  $|x - 1| + |x - 2| + |x - 3| \geq 6$

Consider  $f(x) = |x - 1| + |x - 2| + |x - 3|$

$$f(x) = \begin{cases} 6 - 3x, & x < 1 \\ 4 - x, & 1 \leq x < 2 \\ x, & 2 \leq x < 3 \\ 3x - 6, & x \geq 3 \end{cases}$$

NOTE THIS STEP :



Graph of  $f(x)$  shows  $f(x) \geq 6$  for  $x \leq 0$  or  $x \geq 4$

14. The coefficient of the term independent of  $x$  in the expansion of  $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$  is

- (a)  $\frac{5}{4}$  (b)  $\frac{7}{4}$  (c)  $\frac{9}{4}$  (d) none of these

**Ans. a**, The  $(r+1)$  th term in the expansion of  $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$  is given by

$$T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{3}{2x^2}\right)^r = {}^{10}C_r \frac{x^{5-(r/2)}}{3^{5-(r/2)}} \cdot \frac{3^r}{2^r x^{2r}}$$

$$= {}^{10}C_r \frac{3^{\left(\frac{3r}{2}\right)-5}}{2^r} x^{5-(5r/2)}$$

For  $T_{r+1}$  to be independent of  $x$ , we must have  $5 - (5r/2) = 0$  or  $r = 2$  thus, the 3<sup>rd</sup> term is independent of  $x$  and is equal to

$${}^{10}C_2 \frac{3^{2-5}}{2^2} = \frac{10 \times 9}{2} \times \frac{3^{-2}}{4} = \frac{5}{4}$$

15. The value of  $\left\{\frac{3^{2003}}{28}\right\}$ , where  $\{.\}$  denotes the fractional part, is equal to

- (a)  $\frac{15}{28}$       (b)  $\frac{5}{28}$       (c)  $\frac{19}{28}$       (d)  $\frac{9}{28}$

**Ans. c,**  $3^{2003} = 3^{2001} \cdot 3^2 = 9(27)^{667} = 9(28-1)^{667}$   
 $= 9({}^{667}C_{028} - {}^{667}C_1)(28)^{666} + \dots + {}^{667}C_{667}(-1)^{667}$

That means if we divide  $3^{2003}$  by 28, remainder is 19.

Thus  $\left\{\frac{3^{2003}}{28}\right\} = \frac{19}{28}$

16. If  $(5+2\sqrt{6})^n = I + f$ ;  $I \in \mathbb{N}$ ; and  $0 \leq f < 1$ , then  $I$  equals

- (a)  $\frac{1}{f} - f$       (b)  $\frac{1}{1+f} - f$       (c)  $\frac{1}{1+f} + f$       (d)  $\frac{1}{1-f} - f$

**Ans. d,** Let  $f' = (5-2\sqrt{6})^n$

So  $I + f + f' = (5+2\sqrt{6})^n + (5-2\sqrt{6})^n = \text{an integer}$   
 $\Rightarrow f + f' = 1$

Now  $(I + f) \times f' = (5+2\sqrt{6})^n \times (5-2\sqrt{6})^n = 1$

$\therefore I = \frac{1}{f'} - f = \frac{1}{1-f} - f$

17. The number formed by last two digits of the number  $(17)^{256}$  is

- (a) 81      (b) 80      (c) 91      (d) 93

**Ans. a,**  $(17)^{256} = (289)^{128}$   
 $= (300-11)^{128}$   
 $= {}^{128}C_0(-11)^{128} + 100m$  for some integer  $m$   
 $= 11^{128} + 100m$   
 $= (10+1)^{128} + 100m$   
 $= {}^{128}C_0 1^{128} + {}^{128}C_1 10 + 100m_1 + 100m$  for some integer  $m_1$   
 $= 1 + 1280 + 100k$ ,  $m+m_1 = k$   
 $= 1281 + 100k$

Hence the required number is 81

18. Let  $f: [-10, 10] \rightarrow \mathbb{R}$ , where  $f(x) = \sin x + [x^2/a]$  and  $[.]$  denotes the greatest integer function be an odd function. Then set of values of parameter 'a' is /are

- (a)  $(-10, 10) - \{0\}$       (b)  $(0, 10)$       (c)  $[100, \infty)$       (d)  $(100, \infty)$

**Ans. d,** Since  $f(x)$  is an odd function,

$\left[\frac{x^2}{a}\right] = 0$  for all  $x \in [-10, 10] \Rightarrow 0 \leq \frac{x^2}{a} < 1$  for all  $x \in [-10, 10]$

$\Rightarrow a > 100$

Hence, (d) is the correct answer

19. If  $f$  is a function such that  $f(0) = 2$ ,  $f(1) = 3$  and  $f(x+2) = 2f(x) - f(x+1)$  for every real  $x$  then  $f(5)$  is

- (a) 7      (b) 13      (c) 1      (d) 5

**Ans. b,**  $x = 0 \Rightarrow f(2) = 2f(0) - f(1) = 2 \times 2 - 3 = 1$

$x = 1 \Rightarrow f(3) = 6 - 1 = 5$

$x = 2 \Rightarrow f(4) = 2f(2) - f(3) = 2 \times 1 - 5 = -3$

$x = 3 \Rightarrow f(5) = 2f(3) - f(4) = 2(5) - (-3) = 13$

Hence (b) is correct answer.

20. The domain of definition of

$$f(x) = \sqrt{\log_{0.4} \left( \frac{x-1}{x+5} \right)} \times \frac{1}{x^2-36} \text{ is}$$

- (a)  $(-\infty, 0) \sim \{-6\}$       (b)  $(0, \infty) \sim \{1, 6\}$       (c)  $(1, \infty) \sim \{6\}$       (d)  $(1, \infty] \sim \{6\}$ .

**Ans. c,** For f to be defined  $x \neq -6, 6$  and  $\log_{0.4} \left( \frac{x-1}{x+5} \right) \geq 0, \frac{x-1}{x+5} > 0$ . Since  $\log_a x$  for  $0 < a < 1$  is a decreasing function, we have  $\frac{x-1}{x+5} \leq 1$  and  $\frac{x-1}{x+5} > 0$ . For  $x > -5$ , we must have  $x - 1 \leq x + 5$  and  $x - 1 > 0$ . The first inequality is always true, so we must have  $x > 1$  for  $x < -5$ .

We have  $x - 1 \geq x + 5, x - 1 < 0$ . These inequalities are not possible.

$\Rightarrow$  the domain of f is  $(1, \infty) \sim \{6\}$ .

Hence (c) is the correct answer.

21. Between two junction stations A and B, there are 12 intermediate stations. The number of ways in which a train can be made to stop at 4 of these stations so that no two of these halting stations are consecutive, is

- (a)  ${}^8C_4$       (b)  ${}^9C_4$       (c)  ${}^{12}C_4 - 4$       (d) none of these

**Ans. b,** Let  $x_1$  be the number of stations before the first halting station,  $x_2$  between first and second,  $x_3$  between second and third,  $x_4$  between third and fourth and  $x_5$  on the right of 4<sup>th</sup> stations. Then  $x_1 \geq 0, x_5 \geq 0, x_2, x_3, x_4 \geq 1$  satisfying  $x_1 + x_2 + x_3 + x_4 + x_5 = 8$  .....(i)

The total number of ways is the number of solution of the above equation

Let  $y_1 = x_2 - 1, y_3 = x_3 - 1, y_4 = x_4 - 1$ .

Then (i) reduces to  $x_1 + y_2 + y_3 + y_4 + x_5 = 5$ , where  $y_2, y_3, y_4 \geq 0$ .

The number of solution of this equation is  ${}^{5+5-1}C_{5-1} = {}^9C_4$ .

22. The number of ways of choosing a committee of 2 women and 3 men from 5 women and 6 men, if Mr. A refuses to serve on the committee if Mr. B is a member and Mr. B can only serve, if Miss C is the member of the committee, is

- (a) 60      (b) 84      (c) 124      (d) none of these

**Ans. c,**

**(i) Miss C is taken**

(a) B included  $\Rightarrow$  A excluded  $\Rightarrow {}^4C_1 \cdot {}^4C_2 = 24$

(b) B excluded  $\Rightarrow {}^4C_1 \cdot {}^5C_3 = 40$

**(ii) Miss C is not taken**

$\Rightarrow$  B does not come ;  ${}^4C_2 \cdot {}^5C_3 = 60 \Rightarrow$  Total = 124

23. The number of words that can be formed out of the letters of the word COMMITTEE is -

- (a)  $\frac{9!}{(2!)^3}$       (b)  $\frac{9!}{(2!)^2}$       (c)  $\frac{9!}{2!}$       (d) 9!

**Ans. a,** There are 9 letters in the given word in which two T's, two M's and two E's are identical. Hence the required number of words =  $\frac{9!}{2!2!2!} = \frac{9!}{(2!)^3}$

24. In how many ways can 5 prizes be distributed among 4 boys so that every boy can take one or more prizes?

- (a) 1024      (b) 625      (c) 120      (d) 600

**Ans. a,** First prize may be given to any one of the 4 boys, hence first prize can be distributed in 4 ways. Similarly every one of second, third, fourth and fifth prizes can also be given in 4 ways.

∴ the number of ways of their distribution

$$= 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$$

25. The equation of the straight line equally inclined to the axes and equidistant from the points (1, -2) and (3, 4) is

(a)  $x + y + 1 = 0$     (b)  $x + y + 2 = 0$     (c)  $x - y - 2 = 0$     (d)  $x - y - 1 = 0$

**Ans. d.** Middle point of the line joining points (1, -2) and (3, 4) is (2, 1) which lie on line  $x - y - 1 = 0$ , which is equally inclined to the axes and is at equal distance from the given points.

Hence correct answer is (d)

26. Image of the point P(1, 5) with respect to the line  $4x + 3y + 6 = 0$  is

(a)  $(\frac{19}{25}, \frac{13}{25})$     (b)  $(\frac{13}{25}, \frac{19}{25})$     (c)  $(-7, -1)$     (d)  $(\frac{191}{25}, \frac{113}{25})$

**Ans. c.** If  $Q(h, k)$  is the image then

$$\frac{h-1}{4/5} = \frac{k-5}{3/5} = -10 \Rightarrow h = -7, k = -1$$

27. If  $0^\circ < \theta < 180^\circ$  then

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + \dots + \sqrt{2(1 + \cos \theta)}}}}$$
, there being n number of 2's, is equal to

(a)  $2\cos \frac{\theta}{2^n}$     (b)  $2\cos \frac{\theta}{2^{n-1}}$     (c)  $2\cos \frac{\theta}{2^{n+1}}$     (d) none of these

**Ans. a,**

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + \dots + \sqrt{2(1 + \cos \theta)}}}}$$
, there being n number of 2's

$$= \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots + \sqrt{2(1 + \cos \theta / 2)}}}}$$
, there being (n-1) number 2's

$$= \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots + \sqrt{2(1 + \cos \theta / 2^2)}}}}$$
, there being (n - 2) number 2's

$$= \sqrt{2 \left(1 + \cos \frac{\theta}{2^{n-1}}\right)} = 2 \cos \frac{\theta}{2^n}$$

28. The value of  $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{3\pi}{7}$  is

(a)  $1/8$     (b)  $-1/8$     (c)  $1$     (d)  $0$

**Ans. a,**  $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{3\pi}{7} = \cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \left(\pi - \frac{4\pi}{7}\right) = -\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7}$

$$= -\left[\frac{\sin(2^3 \cdot \frac{\pi}{7})}{2^3 \sin \frac{\pi}{7}}\right] = -\frac{\sin \frac{8\pi}{7}}{8 \sin \frac{\pi}{7}} = \frac{1}{8}$$

29. The general solution of the equation

$$(\sqrt{3} - 1) \sin \theta + (\sqrt{3} + 1) \cos \theta = 2$$
 is

(a)  $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$     (b)  $n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{12}$     (c)  $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{12}$     (d)  $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{12}$

**Ans. a,** Dividing by  $\sqrt{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2} = 2\sqrt{2}$

$$\frac{\sqrt{3}-1}{2\sqrt{2}} \sin \theta + \frac{\sqrt{3}+1}{2\sqrt{2}} \cos \theta = \frac{1}{\sqrt{2}}, \quad \sin \frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\sin \frac{\pi}{12} \sin \theta + \cos \frac{\pi}{12} \cos \theta = \frac{1}{\sqrt{2}}$$

$$\cos\left(\theta - \frac{\pi}{12}\right) = \frac{1}{\sqrt{2}}$$

$$= \cos\left(2n\pi \pm \frac{\pi}{4}\right)$$

$$\Rightarrow \theta - \frac{\pi}{12} = 2n\pi \pm \frac{\pi}{4}$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$$

30. The general solution of the equation

$$\cos x \cdot \cos 6x = -1 \text{ is}$$

(a)  $x = (2n+1)\pi, n \in I$     (b)  $x = 2n\pi, n \in I$     (c)  $x = (2n-1)\frac{\pi}{2}, n \in I$     (d) none of these

**Ans. a,** We have  $\cos x \cdot \cos 6x = -1$

$$2\cos x \cdot \cos 6x = -2$$

$$\cos 7x \cdot \cos 5x = -2$$

Which is possible by when

$$\cos 7x = -1 \text{ \& \& } \cos 5x = -1$$

The values of  $x$  satisfying these two equation simultaneously and lying between 0 and  $2\pi$  is  $\pi$ .

Therefore the general solution is  $(2n+1)\pi$



## Answer key

Ques.	1	2	3	4	5	6	7	8	9	10
Ans.	a	d	a	b	d	c	c	b	d	a
Ques.	11	12	13	14	15	16	17	18	19	20
Ans.	a	a	c	a	c	d	a	d	b	c
Ques.	21	22	23	24	25	26	27	28	29	30
Ans.	b	c	a	a	d	c	a	a	a	a