Solution of Paper held on 15.09.17 (Dropper Mathematics)

1. If the roots of the equation $ax^2+bx+c=0$ are of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$, then value of $(a+b+c)^2$ is
(a) $2b^2-ac$ (b) b^2-2ac (c) b^2-4ac (d) $4b^2-2ac$

Solution: (c) By hypothesis $\frac{\alpha}{\alpha-1}+\frac{\alpha+1}{\alpha}=-\frac{b}{a}$ and $\frac{\alpha}{\alpha-1}.\frac{\alpha+1}{\alpha}=\frac{c}{a}$

$\Rightarrow \frac{2\alpha^2-1}{\alpha^2-\alpha}=-\frac{b}{a}$ and $\alpha=\frac{c+a}{c-a}$

$\Rightarrow (c+a)^2 2b(c+a)+b^2=b^2-4ac$

$\Rightarrow (a+b+c)^2=b^2-4ac$

2. The value of a, for which one root of the equation $(a-5)x^2-2ax+(a-4)=0$ is smaller than 1 and the other is greater than 2 is
(a) $a\in(5,24)$ (b) $a\in\left(\frac{20}{3},\infty\right)$ (c) $a\in(5,\infty)$ (d) $(-\infty,\infty)$

Solution: (a) (i) $D>0$, $4a^2-4(a-5)(a-4)>0$

$9a-20>0 \Rightarrow a>\frac{20}{9} \Rightarrow a\in\left(\frac{20}{9},\infty\right)$ (i)

(ii) $(a-5)f(1)<0$; $(a-5)f(2)<0$

$\Rightarrow (a-5)(a-5-2a+a-4)<0$

$\Rightarrow a>5 \Rightarrow a\in(5,\infty)$ (ii)

and $(a-5)(a-24)<0 \Rightarrow 5<a<24$

$\Rightarrow a\in(5,24)$ (iii)

Using (1), (ii) & (iii)

The common condition is $a\in(5,24)$

3. $\sin ax+\cos ax$ and $|\sin x|+|\cos x|$ are periodic of same fundamental period, if a equals
[a] 0 [b] 1 [c] 2 [d] 4

Solution :- [d]

Period of $\sin ax$ is $\frac{2\pi}{a}$

And period of $\cos a\,x$ is $\frac{2\pi}{a}$

$\therefore$ Period of $\sin ax+\cos ax$ is $\frac{2\pi}{a}$

And period of $|\sin x|+|\cos x|$ is $\frac{\pi}{2}$

Given, $\frac{2\pi}{a}=\frac{\pi}{2}$

$\Rightarrow$ $a=4$

4. If $f(2x+3y,2x-7y)=20x$, then $f(x,y)$ equals
[a] $7x-3y$ [b] $7x+3y$ [c] $3x-7y$ [d] $3x+7y$

Solution :- [b]

Let $2x+3y=A$ and $2x-7y=B$

Then, $7A+3B=20x$

$\therefore$ $f(A,B)=7A+3B$

$\therefore$ $f(x,y)=7x+3y$

5. The value of $\lim_{x\to0}\frac{(1+x)^{1/x}-e+\frac{1}{2}ex}{x^2}$ is
[a] $\frac{11}{24}e$ [b] $-\frac{11e}{24}$ [c] $\frac{e}{24}$ [d] none of these

Solution :- [a]

$\lim_{x\to0}\frac{(1+x)^{1/x}-e+\frac{1}{2}ex}{x^2}$

$=\lim_{x\to0}\frac{e\left(1-\frac{x}{2}+\frac{11}{24}x^2+\cdots\right)-e+\frac{1}{2}ex}{x^2}$

$=\lim_{x\to0}\frac{\frac{11}{24}e\,x^2+\cdots}{x^2}=\frac{11}{24}e$

6. The solution of the equation $3^{\log_a x}+3x^{\log_a 3}=2$ is given by
[a] $a^{\log_3 a}$ [b] $\left(\frac{2}{a}\right)^{\log_3 2}$ [c] $a^{-\log_3 2}$ [d] $2^{-\log_3 a}$

Solution :-

$\Rightarrow 3^{\log_a x}+3.x^{\log_a 3}=2$

$\Rightarrow$ $3^{\log_a x}+3.3^{\log_a x}=2$

$\Rightarrow$ $4.3^{\log_a x}=2$

$\Rightarrow$ $3^{\log_a x}=\left(\frac{1}{2}\right)$

$\Rightarrow$ $\log_a x=-\log_3 2 \Rightarrow x=a^{-\log_3 2}=a^{\log_3(2^{-1})}$

$=(2^{-1})^{\log_3 a}=2^{-\log_3 a}$

7. Let $f(n)=2\cos nx$, $\forall n\in N$, then $f(1)f(n+1)-f(n)$ is equal to
(a) $f(n+3)$ (b) $f(n+2)$ (c) $f(n+1)f(2)$ (d) $f(n+2)f(2)$

SOLUTION : (B) $f(n)=2\cos nx$

$\Rightarrow f(1)f(n+1)-f(n)$

$=4\cos xc\cos(n+1)x-2\cos nx$

$=2[2\cos(n+1)x\cos x-\cos nx]$

$=2[\cos(n+2)x+\cos nx+\cos nx]$

$=2\cos(n+2)x=f(n+2)$

8. The value of $\lim_{|x|\to\infty}\cos(tan^{-1}(\sin(tan^{-1}x)))$ is
(a) -1 (b) $\sqrt{2}$ (c) $-\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

SOLUTION : (D) $\lim_{|x|\to\infty}\cos(tan^{-1}(\sin(tan^{-1}x)))$

$=\cos(tan^{-1}(\sin(tan^{-1}\infty)))$

$=\cos(tan^{-1}(\sin(\pi/2)))$

$=\cos(tan^{-1}(1))=\cos\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

9. In any ΔABC, if $\cot\frac{A}{2},\cot\frac{B}{2},\cot\frac{C}{2}$ are in A.P., then a, b, c are in
(a) A.P. (b) G.P. (c) H.P. (d) none of these

SOLUTION : (A) $\cot\frac{A}{2},\cot\frac{B}{2},\cot\frac{C}{2}$ are in A.P.

$\Rightarrow 2\cot\frac{B}{2}=\cot\frac{A}{2}+\cot\frac{C}{2}$

$\Rightarrow 2\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$

$=\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}+\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$\Rightarrow 2(s-b)=s-a+s-c$

$\Rightarrow 2b=a+c$

$\Rightarrow a,b,c$ are in A.P.

10. If $(b^2-4ac)^2(1+4a^2)<64a^2, a<0$, then maximum value of quadratic expression ax^2+bx+c is always less than
(a) 0 (b) 2 (c) -1 (d) -2

SOLUTION : (B) $\frac{(b^2-4ac)^2}{16a^2}<\frac{4}{1+4a^2}$

Now,

$\max(ax^2+bx+c)=-\frac{b^2-4ac}{4a}$

also,

$\frac{-2}{\sqrt{1+4a^2}}<\frac{b^2-4ac}{4a}<\frac{2}{\sqrt{1+4a^2}}$ [from (1)]

So maximum value is always less than 2(when $a\to0$).

11. The number of integral values of x satisfying $\sqrt{-x^2+10x-16}<x-2$ is
(a) 0 (b) 1 (c) 2 (d) 3

SOLUTION : (D) $\sqrt{-x^2+10x-16}<x-2$

We must have

$-x^2+10x-16\ge0$

$\Rightarrow x^2-10x+16\le0$

$\Rightarrow 2\le x\le8$ (1)

Also,

$-x^2+10x-16<x^2-4x+4$

$\Rightarrow 2x^2-14x+20>0$

$\Rightarrow$ $x^2-7x+10>0$

$\Rightarrow x^2-7x+10>0$

$\Rightarrow x>5$ or $x<2$

From (1) and (2)

$5<x\le8 \Rightarrow x=6,7,8$

12. If $z=x+iy$ and $x^2+y^2=16$, then the range of $||x|-|y||$ is
(a) [0, 4] (b) [0, 2] (c) [2, 4] (d) none of these

SOLUTION : (A) here $x=4\cos\theta, y=4\sin\theta$.

$\therefore ||x|-|y||=|4|\cos\theta|-4|\sin\theta||$

$=4||\cos\theta|-|\sin\theta||$

$=4\sqrt{1-2|\cos\theta||\sin\theta|}$

$=4\sqrt{1-|\sin2\theta|}$

Hence, the range is [0, 4].

13. z and z_2 are two distinct points in an argand plane. If $a|z_1|=b|z_2|$ (where $a,b\in R$), then the point $(az_1/bz_2)+(bz_2/az_1)$ is a point on the
(a) Line segment [-2, 2] of the real axis (b) line segment [-2, 2] of the imaginary axis
(b) Unit circle |z|=1 (d) the line with $\arg z=tan^{-1}2$

SOLUTION : (A) assuming $\arg z_1=\theta$ and $\arg z_2=\theta+a$,

$\frac{az_1}{bz_2}+\frac{bz_2}{az_1}=\frac{a|z_1|e^{i\theta}}{b|z_2|e^{i(\theta+\alpha)}}+\frac{b|z_2|e^{i(\theta+\alpha)}}{a|z_1|e^{i\theta}}$

$=e^{i\alpha}+e^{i\alpha}=2\cos\alpha$

Hence, the point lies on the line segment [-2, 2] of the real axis.

14. The number of ordered pairs of integer (x, y) satisfying the equation $x^2+6x+y^2=4$ is
(a) 2 (b) 8 (c) 6 (d) none of these

SOLUTION : (B). $(x+3)^2+y^2=13$

$\Rightarrow x+3=\pm2, y=\pm3$ or $x+3=\pm3, y=\pm2$

15. Let a and b represent the length of a right triangle's legs. If d is the diameter of a circle inscribed into the triangle m and D is the diameter of a circle circumscribed on the triangle, then $d+D$ equals
(a) $a+b$ (b) $2(a+b)$ (c) $\frac{1}{2}(a+b)$ (d) $\sqrt{a^2+b^2}$

SOLUTION : (A) $AB=\sqrt{a^2+b^2}$

Hence, D = $\sqrt{b^2+a^2}$ (1)

Now, $\frac{d}{2}=\frac{\Delta}{s}=\frac{ab}{2s}$ [where s is semi- perimeter]

$\therefore \frac{d}{2}=\frac{ab}{a+b+\sqrt{a^2+b^2}}$

Or $d=\frac{2ab}{a+b+\sqrt{a^2+b^2}}$ (2)

From eqs. (1) and (2)

$d+D=\frac{\sqrt{a^2+b^2}\left[(a+b)+\sqrt{a^2+b^2}\right]+2ab}{a+b+\sqrt{a^2+b^2}}$

$=\frac{(a+b)^2+(a+b)\sqrt{a^2+b^2}}{a+b+\sqrt{a^2+b^2}}=a+b$

16. If $f(3x+2)+f(3x+29)=0\ \forall x\in R$, then the period of $f(x)$ is
(a) 7 (b) 8 (c) 10 (d) none of these

SOLUTION : (D) $f(3x+2)+f(3x+29)=0$ (1)

Replacing x by $x+9$, we get

$f(3(x+9)+2)+f(3(x+9)+29)=0$

$\Rightarrow f(3x+29)+f(3x+56)=0$ (2)

From (1) and (2), we get

$f(3x+2)=f(3x+56)$

$\Rightarrow f(3x+2)=f(3(x+18)+2)$

$\Rightarrow f(x)$ is periodic with period 18.

17. If $f(x)=\begin{cases}\frac{1-\sin x}{(\pi-2x)^2}.\frac{\log\sin x}{\log(1+\pi^2-4\pi x+4x^2)}; & x\ne\frac{\pi}{2}\\ c; & x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$, then c equals:
(a) $-\frac{1}{64}$ (b) $-\frac{1}{32}$ (c) $-\frac{1}{28}$ (d) $-\frac{1}{16}$

Solution: (a)

For $f(x)$ to be continuous we must have $lim_{x\to\frac{\pi}{2}}f(x)=f\left(\frac{\pi}{2}\right)$.

Now $lim_{x\to\frac{\pi}{2}}f(x)$

$=lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{(\pi-2x)^2}.\frac{\log\sin x}{\log(1+\pi^2-4\pi x+4x^2)}$

$=lim_{h\to0}\frac{1-\sin\left(\frac{\pi}{2}+h\right)}{(-2h)^2}.\frac{\log\sin\left(\frac{\pi}{2}+h\right)}{\log(1+4h^2)}$

$\left[Putting\ x=\frac{\pi}{2}+h\ so\ that\ h\to0\ as\ x\to\frac{\pi}{2}\right]$

$=lim_{h\to0}\frac{1-\cos h}{4h^2}.\frac{\log\cos h}{\log(1+4h^2)}$

$=lim_{h\to0}\frac{1-\cos h}{4h^2}\times\frac{\log(1+\cos h-1)}{\cos h-1}\times\frac{4h^2}{\log(1\times4h^2)}\times\frac{\cos h-1}{4h^2}$

$=lim_{h\to0}\left(\frac{1-\cos h}{4h^2}\right)^2\frac{\log(1+(\cos h-1))}{\cos h-1}\times\frac{4h^2}{\log(1+4h^2)}$

$=lim_{h\to0}\left(\frac{\sin^2\frac{h}{2}}{2h^2}\right)^2\frac{\log(1+(\cos h-1))}{\cos h-1}\times\frac{4h^2}{\log(1+4h^2)}$

$=-\frac{1}{64}lim_{h\to0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\frac{\log(1+(\cos h-1))}{\cos h-1}\times\frac{4h^2}{\log(1+4h^2)}$

$=-\frac{1}{64}(1)^4.\log e.\frac{1}{\log e}=-\frac{1}{64}$

Thus $-\frac{1}{64}=c \Rightarrow c=-\frac{1}{64}$

18. For a real number x, let [x] denote the greatest integer less than or equal to x. Then $f(x)=\frac{\tan(\pi[x-\pi])}{1+[x]^2}$ is:
(a) Continuous at some x
(b) Continuous at all x but f (x) does not exist
(c) F(x) exists for all x, but f(x) does not exist
(d) $f(x)$ exists for all x.

Solution: (d)

Since $f(x)=0$ for all x, $\therefore f(x)=0$, for all x.

$[\because \tan(\pi[x-\pi])=\tan n\pi$, where n is an integer as $[x-\pi]=$ an integer $n]$

19. Let $f(x)=\frac{\sin4\pi[x]}{1+[x]^2}$, where $[x]$ is the greatest integer $\le x$, then:
(a) $f(x)$ is not differentiable at some points
(b) $f(x)$ exists but is different from 0
(c) $f(x)=0$ for all x
(d) $f(x)=0$ but f is not a constant function.

Solution: (c)

Clearly $[x]^2\ne1$, and $\sin4\pi[x]=0, \forall x$

$\therefore f(x)=0 \Rightarrow f(x)=0$ for all x.

20. Let f be a function satisfying $f(x+y)=f(x)+f(y)$ and $f(x)=x^2g(x)$, for all x and y, where $g(x)$ is a continuous function. Then $f(x)$ is:
(a) g' (x) (b) g(0) (c) g(0) + g(0) (d) 0.

Solution: (d)

$f'(x)=lim_{\delta x\to0}\frac{f(x+\delta x)-f(x)}{\delta x}$

$=lim_{\delta x\to0}\frac{f(x)+f(\delta x)-f(x)}{\delta x}$

$[\because f(x+y)=f(x)+f(y)]$

$=lim_{\delta x\to0}\frac{f(\delta x)}{\delta x}=lim_{\delta x\to0}\frac{(\delta x)^2g(\delta x)}{\delta x}$

$[\because f(x)=x^2g(x)]$

$=lim_{\delta x\to0}\delta x\,g(\delta x)=0\times g(0)=0.$

21. The value of $lim_{x\to0}(\cos x+a\sin bx)^{1/x}$ is:
(a) 1 (b) ab (c) e^{ab} (d) $e^{b/a}$.

Solution: (c)

$lim_{x\to0}(\cos x+a\sin bx)^{\frac{1}{x}}$

$=lim_{x\to0}\left[\left(1-\frac{x^2}{2!}+\ldots\right)+a(bx-\ldots\ldots)\right]^{\frac{1}{x}}$

$=lim_{x\to0}\left[(1+abx)^{\frac{1}{abx}}\right]^{ab}=e^{ab}.$

22. Y = $1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\cdots+$ to ∞ with $|x|>1$, then $\frac{dy}{dx}$
(a) $\frac{x^2}{y^2}$ (b) x^2y^2 (c) $\frac{y^2}{x^2}$ (d) $-\frac{y^2}{x^2}$

Solution: Ans.: (d)

$\therefore$ $|x|>1$

$\therefore$ $\frac{1}{|x|}<1$ or $-1<\frac{1}{x}<1$

$\therefore$ $y=\frac{1}{1-\frac{1}{x}}=\frac{x}{x-1}$ (1)

$\Rightarrow$ $\frac{dy}{dx}=-\frac{1}{(x-1)^2}=-\left(\frac{y}{x}\right)^2$

23. If the function $f(x)=x^3+e^{x/2}$ and $g(x)=f^{-1}(x)$, then the value of g'(1) equals:
(a) 1 (b) 2 (c) 3 (d) 4

Solution: Ans.: (b)

$\because$ $f(0)=1; f'(x)=3x^2+\frac{1}{2}e^{x/2}$

and $g(x)=f^{-1}(x)$

we have $f(g(x))=x$

Put x = 1, then

f'(g(1)) . g'(1) = 1

$\Rightarrow$ $g'(1)=\frac{1}{f'(g(1))}=\frac{1}{f'(0)}=2$ $(\because f(0)=1 \Rightarrow g(1)=0)$

24. Let y be an implicit function of x defined by $x^{2x}-2x^x\cot y-1=0$, then y'(1) equals:
(a) -1 (b) 1 (c) $\log_e 2$ (d) $-\log_e 2$

Solution: Ans.: (a)

$\because$ $x^{2x}-2x^x\cot y-1=0$ (1)

$\Rightarrow$ $2x^x\cot y=x^{2x}-1$

$\Rightarrow$ $\cot y=\left(\frac{x^x-x^{-x}}{2}\right)$

$\therefore$ $-cosec^2y\frac{dy}{dx}=\frac{x^x(1+log_e x)+x^{-x}(1+log_e x)}{2}$

$\Rightarrow$ $-cosec^2y\frac{dy}{dx}=\left(\frac{x^x-x^{-x}}{2}\right)(1+log_e x)$

. $-cosec^2y\ .\ y'(x)=\left(\frac{x^x-x^{-x}}{2}\right)(1+log_e x)$

Or $y'(1)=-\sin^2\left(\frac{\pi}{2}\right)$

$=-1$

(from (i), at x = cot y = 0 $\therefore y=\frac{\pi}{2}=-1$)

25. $\frac{d^2x}{dy^2}$ equals:
a. $\left(\frac{d^2x}{dx^2}\right)^{-1}$ (b) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-1}$ (c) $\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$ (d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Solution: (d)

$\frac{d^2x}{dy^2}=\frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{d}{dy}\left(\left(\frac{dy}{dx}\right)^{-1}\right)$

$=\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^{-1}\right).\frac{dx}{dy}=\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^{-1}\right).\left(\frac{dy}{dx}\right)^{-1}$

$=(-1)\left(\frac{dy}{dx}\right)^{-2}\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^{-1}=-\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^{-3}.$

26. if f(x) and g(x) are two differentiable functions on [0, 2] such that f''(x) – g''(x) = 0, f'(1) = 2, g'(1) = 4, f(2) = 3, g(2) = 9, then f(x) – g(x) at x = 3/2 is: **[AIEEE 2002]**
(a) 0 (b) 2 (c) 10 (d) -5

Solution: Ans.: (d)

f''(x) –g''(x) = 0

Integrating w.r.t. x,

f'(x) – g'(x) = c

At x = 1,

f'(1) – g'(1) = c

$\Rightarrow$ $2-4=c$

$\Rightarrow$ $c=-2$

Hence, f'(x) - g'(x) = -2

Again integrating w.r.t. x,

f(x) – g(x) = -2x + c_1

At x = 2,

$\Rightarrow$ 3 – 9 + 4 = c_1

$\Rightarrow$ c_1 = -2

Then f(x) – g(x) = -2x – 2 = -(2x + 2)

$\therefore$ $f\left(\frac{3}{2}\right)-g\left(\frac{3}{2}\right)=-\left(2\times\frac{3}{2}+2\right)=-5$

27. If f(x) is twice differentiable polynomial function such that f(1) = 1, f(2) = -4, f(3) = 9, then:
(a) F''(x) = 2, $\forall x\in R$
(b) There exist at least one x$\in$ (1,3) such that f''(x) = 2
(c) There exist at least one x$\in$ (2,3) such that f'(x) = 5 = f''(x)
(d) There exist at leased one x$\in$ 3.

Solution: Ans.: (b)

Let a function be g(x) = f(x) – x^2

$\Rightarrow$ g(x) has at least 3 real roots which are x = 1, 2, 3

$\Rightarrow$ g'(x) has atleast 2 real roots in x$\in$ (1, 3)

$\Rightarrow$g''(x) has atleast 1 real roots in x$\in$ (1, 3)

$\Rightarrow$ f'(x) = 2 for atleast one x$\in$(1, 3)

28. If $f(x+y)=f(x)f(y)$ and $f(x)=1+xg(x)G(x)$, where $lim_{x\to a}g(x)=a$ and $lim_{x\to0}G(x)=b$. Then $f'(x)$ is equal to:
(a) $\frac{a}{b}$ (b) $1+ab$ (c) ab (d) None of these

Solution: (d)

$f'(x)=lim_{h\to0}\frac{f(x+h)-f(x)}{h}$

$=lim_{h\to0}\frac{f(x).f(h)-f(x)}{h}$

$[\because f(x+y)=f(x)f(y)]$

$=lim_{h\to0}\frac{f(x)(f(h)-1)}{h}$

$=f(x)\ lim_{h\to0}\frac{1+hg(h)G(h)-1}{h}$

$=f(x)\ lim_{h\to0}g(h)\ lim_{h\to0}G(h)$

$=f(x).(a)=ab\,f(x).$

29. If f is derivable at $x=a$, then $lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ is:
(a) $f(a)-af'(a)$ (b) $af'(a)-f'(a)$ (c) $f'(a)$ (d) None of these

Solution: (a)

$lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$

$=lim_{x\to a}\frac{-a[f(x)-f(a)]+xf(a)-af(a)}{x-a}$

$=-a\ lim_{x\to a}\frac{f(x)-f(a)}{x-a}+f(a)=-af'(a)+f(a).$

30. If $G(x)=\frac{1}{\sqrt{25-x^2}}$, then $lim_{x\to4}\frac{G(x)-G(4)}{x-4}$ has the value:
(a) $\frac{4}{27}$ (b) $-\frac{4}{27}$ (c) $-\frac{4}{3\sqrt{3}}$ (d) $\frac{4}{9}$

Solution: (a)

Reqd. limit = G'(4)

i.e. $\frac{d}{dx}\left[(25-x^2)^{-1/2}\right]$ at $x=4$

$=\left(-\frac{1}{2}\frac{1}{(25-x^2)^{3/2}}(-2x)\right)_{at\,x=4}$

$=\frac{4}{(25-16)^{3/2}}$

$=\frac{4}{(9)^{3/2}}=\frac{4}{(3^2)^{3/2}}=\frac{4}{3^3}=\frac{4}{27}.$