Mathematics solution paper

Class XII Comp.

1. Ans. B, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ applying $c_1 \to c_1 + c_2 + c_3$

   $= \begin{vmatrix} 3 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix} = 3(\omega^2 - \omega) = 3\omega(\omega - 1)$

2. Ans. D, if c = 2cosθ Expanding along with R,

   $$\Delta = C(c^2 - 1) - 1(c - 6) + 0(1 - 6c)$$
   $$= c^3 - 2C + 6$$
   $$= 8\cos^2\theta - 4\cos\theta + 6$$

3. Ans. C, we have, A = $\begin{bmatrix} i & 0 \\ 0 & i^{4n} \end{bmatrix}$

   Clearly, A is a diagonal matrix. Therefore, A^{4n} is also a diagonal matrix such that

   $$A^{4n} = \begin{bmatrix} i^{4n} & 0 \\ 0 & i^{4n} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

4. Ans. A, $\because |KA| = k^3|n|$, K is a scaler and A is of order $n \times n$, Here A is of order 3×3 and k = 3

5. Ans. C, f(x) is to be defined when $x^2 - 1 > 0$ and 3 + x > 0 and 3 + x $\neq$ 1, i.e.,

   $x^2 > 1$ *and* $x > -3$ and $x \neq -2$

   i.e., x < -1 or x > 1 and x > -3 and $x \neq -2$

   $\therefore D_f$ = (-3, -2)∪(-2, -1)∪(1,∞)

6. Ans. A, we have f(x) = $\left[\log_{10}\left(\frac{5x - x^2}{4}\right)\right]^{1/2}$ (1)

   From (1), clearly, f(x) is defined for those values of x for which

   $\log_{10}\left[\frac{5x - x^2}{4}\right] \geq 0$

   or $\left(\frac{5x - x^2}{4}\right) \geq 10^0$

   or $\left(\frac{5x - x^2}{4}\right) \geq 1$

   or $x^2 - 5x + 4 \leq 0$

   or(x - 1)(x - 4) $\leq$ 0

   Hence, the domain of the function is [1,4].

7. Ans. A, Since f(x) and f^{-1}(x) are symmetric about the line y = -x, if (a, β) lies on y = f(x), then (-β, - a) lies on y = f^{-1}. Therefore, (-a,-β) lies on y =f(x).

   Hence, y =f(x) is odd.

8. Ans. C, let x, y ∈ N such that f(x) = f(y). then.

   f(x) = f(y)

   or x^2 + y + 1 = y^2 + y + 1

   or(x - y)(x + y + 1) = 0

   i.e., x = y or x = (-y -1) ∉ N

   Therefore, f is one – one.

   Also, f(x) does not take all positive integral values. Hence, f is into.

9. Ans. B, Triangle may have equal area therefore, f is not one-one.

Since each positive real number can represent area of a triangle, f is onto.

10. Ans. C, u(x) = 7v(x) or $u'(x)$ = $7v'$(x) or p = 7 (given)

Again $\frac{u(x)}{v(x)} = 7 \text{ or } \left(\frac{u(x)}{v(x)}\right)'$ = 0 or q = 0

Now, $\frac{p+q}{p-q}$ = $\frac{7+0}{7\_0}$ = 1

11. Ans. A, Y = f(x) is an even function and y = g(x) is an odd function.

or h(x) = f(x) g(x) is an odd function.

or h(x) = -h(-x)

∴ h'(x) = h'(-x)

or h''(x) = -h''(-x)

or h'''(x) = h'''(-x)

Now, we cannot determine the value of h'''(0).

12. Ans. B, We have $\frac{f(2x+2y)}{f(2x-2y)} = \frac{\sin(x+y)}{\sin(x-y)}$

or $\frac{f(\alpha)}{sin\frac{\alpha}{2}}$ = $\frac{f(\beta)}{sin\frac{\beta}{2}}$ = K

or f(x)= K sin$\frac{x}{2}$

∴f'(x) = $\frac{K}{2}$ cos $\frac{x}{2}$

and f''(x) = $\frac{-K}{4}$ sin $\frac{x}{2}$

or 4 f'' (x) + f(x) = 0

13. Ans. C, $\lim_{t\to x} \frac{e^t f(x) - e^x f(t)}{(t-x)((fx))^2}$ = 2

or $\lim_{t\to x} \frac{e^t f(x) - e^x f\prime(t)}{1.((fx))^2}$ = 2

or $\frac{e^x f(x) - e^x f\prime(t)}{(t-x)((fx))^2}$ = 2

or $\frac{d}{dx}\left(\frac{e^x}{f(x)}\right)$= 2

or $\frac{e^x}{f(x)}$ = 2x + c

f(0) = 2

or $\frac{1}{f(0)}$ = c

c = 2

or f(x) = $\frac{e^x}{2x+2}$

or f(x) = $\frac{e^x(2x+2)-2e^x}{(2x+2)^2}$

14. Ans. C, solving y = $|x^2 - 1|$ and y = $\sqrt{7 - x^2}$, we have

$|x^2 - 1|$ = $\sqrt{7 - x^2}$

or $x^4 - 2x^2 + 1 = 7 - x^2$

or $x^4 - x^2 - 6 = 0$

or $(x^2 - 3)(x^2 + 2)$ = 0

or x = $\pm\sqrt{3}$

points of intersection of the curves y =$|x^2 - 1|$ and y = $\sqrt{7 - x^2}$ are $(\pm\sqrt{3}, 2)$.

Since both the curves are symmetrical about the y- axis, points of intersection are also symmetrical.

Now, y = $x^2 - 1$ or $\frac{dy}{dx}$ = 2x

Or $m_1 = \left.\frac{dy}{dx}\right|_{(\sqrt{3},2)}$ =$2\sqrt{3}$

and y = $\sqrt{7 - x^2}$ or $\frac{dy}{dx}$ = - $\frac{x}{y}$

or $m_2 = \left.\frac{dy}{dx}\right|_{(\sqrt{3},2)}$ =- $\frac{\sqrt{3}}{2}$ or tan θ =$\left|\frac{5\sqrt{3}}{4}\right|$

15. Ans. D f(x) = ax^3 + bx^2 + 11x- 6

Satisfies conditions of Rolle's theorm in [1, 3]. Therefore,

f(1) = f(3)

or a + b + 11 – 6 = 27a + 9b + 33 – 6

or 13a + 4b = -11 (1)

and f'(x) = 3ax^2 + 2bx + 11

or f'$\left(2 + \frac{1}{\sqrt{3}}\right)$= 3a$\left(2 + \frac{1}{\sqrt{3}}\right)^2$ + 2b$\left(2 + \frac{1}{\sqrt{3}}\right)$ + 11 = 0

or 3a$\left(4 + \frac{1}{3} + \frac{4}{\sqrt{3}}\right)$ + 2b$\left(2 + \frac{1}{\sqrt{3}}\right)$ + 11 = 0 (2)

From equations (1) and (2), we get a = 1, b = - 6.

16. Ans. D, let g(x) = f (x) - x^2. We have g(1) = 0, g(2) = 0, g(3) = 0

[$\because f(1) = 1, f(2) = 4, f(3) = 9$].

From Rolle's theorem on g(x), g'(x) = 0 for at least

X ∈ (1,2). Let g'(c_1) = 0 where $c_1 \epsilon$ (1,2).

Similarly, g(x) = 0 for at least one x ∈ (2,3). Let $g'(c_2)$ = 0 where

c_2 ∈ (2,3). Therefore,

g' (c_1) = g'(c_2) = 0

by Rolle's theorm, at least one x ∈ (c_1, c_2) such that g''(x) = 0 or f''(x) = 2 for some x ∈ (1,3).

17. Ans. B, f$\left(\frac{5\pi}{6}\right) = \log sin\left(\frac{5\pi}{6}\right)$ = log sin$\frac{\pi}{6}$ = log$\frac{1}{2}$ = - log 2

f$\left(\frac{\pi}{6}\right)$ = log sin$\frac{\pi}{6}$ = - log 2

f'(c) = $\frac{1}{\sin x}$ cos x = cot x

By Lagrange's mean value theorm,

$\frac{f(5\pi / 6) - f(\pi/6)}{(5\pi / 6) - (\pi/6)}$ = cot c

or cot c = 0 or c = $\frac{\pi}{2}$

Thus, c = $\frac{\pi}{2}$ ∈ ($\pi/6, 5\pi/6$).

18. Ans. C, f'(x)=$\frac{0.6(1+x)^{-0.4}(1+x^{0.6})-0.6x^{-0.4}(1+x)^{0.6}}{(1+x^{0.6})^2}$

=0.6$\frac{(1+x^{0.6})-x^{-0.4}(1+x)^1}{(1+x^{0.6})^2(1+x)^{0.4}}$ = 0.6$\frac{(1+x^{0.6})x^{0.4}-(1+x)^1}{(1+x^{0.6})^2(1+x)^{0.4}}$

=0.6$\frac{x^{0.4}-1}{(1+x^{0.6})^2(1+x)^{0.4}x^{0.4}}$ < 0 ∀ x ∈ (0, 1)

Hence, f(x) is decreasing. Thus,

$f(x)_{max} = f(0) = 1$

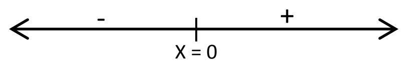
19. Ans. D, We have f(x) = $\frac{x^2 - a}{x^2 + a}$ = 1- $\frac{2a}{x^2+a}$

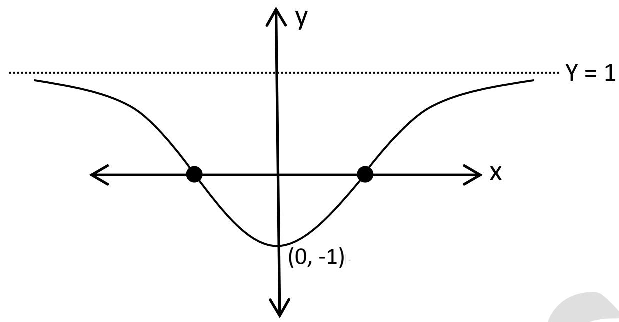
Clearly, range of is [-1,1). Now,

f'(x) = $\frac{4ax}{(x^2+a)^2}$

and $f''(x) = \frac{4a}{(x^2+a)^3}(a - 3x^2)$

sign scheme of f' (x) is as follows;





Thus, f(x) is decreasing on (-∞, 0) and increasing on (0, ∞). Therefore, f(x) has a local minimum at x = 0.

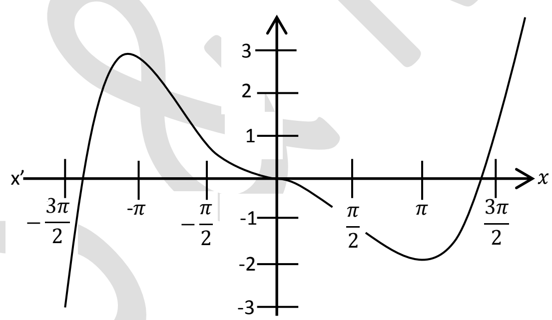
20. Ans. A, $h'(x) = \frac{m}{n}x^{\frac{m-n}{n}} = \frac{m}{n}x^{-\left(\frac{even}{odd}\right)}$

As h' (x) is undefined at x = 0 and h'(x) does not change its sign in the neighborhood, there are no extremums.

21. Ans. D, Let there be a value of k for which $x^3 - 3x + k = 0$ has two distinct roots between 0 and 1. Let a, b be two distinct roots of $x^3 - 3x + k = 0$ lying between 0 and I such that a <b. let $f(x) = x^3 - 3x + k$. Then f(a) = f(b) = 0. Since between any two roots of a polynomial f(x), there exists at least one root of its derivative $f'(x) = 3x^2 - 3$ has at least one roots between a and b. but f'(x) = 0 has two roots equal to ± 1 which do not lie between a and b. Hence, f(x) = 0 has no real roots lying between 0 and 1 for any value of k.

22. Ans. B. f'(x) = - x sin x = 0 when x = 0 or π

$\left.\begin{matrix} f'(0^-) = (-)(-)(-)<0 \\ f'(0^+) = (-)(+)(+)<0 \end{matrix}\right\}$ no sign change



This also implies that f is decreasing at x = 0.

Thus, (b) is correct.

f''(x) = - (x cos x + sin x)

$f''(\pi) = -(-\pi) > 0$, i.e., minima at $x = \pi$

$f''(-\pi) = -(-\pi) < 0$, i.e., minima at $x = -\pi$

23. Ans. B, Let $I = \int \frac{\cos^8 x - \sin^8 x}{1 - 2\sin^2 x \cos^2 x} dx$

$= \int \frac{(\cos^4 x)^2 - (\sin^4 x)^2}{(\cos^2 x + \sin^2 x)^2 - 2\sin^2 x \cos^2 x} dx$

$= \int \frac{(cos^4 x + sin^2 x)(cos^4 x - sin^2 x)}{(cos^4 x + sin^2 x)}$dx

$= \int (cos^4 x - sin^2 x)$dx

$= \int (cos^2 x + sin^2 x)(cos^2 x - sin^2 x)$dx

$= \int (cos^2 x - sin^2 x)$dx = $\int cos2\ x\ dx$

$= \frac{1}{2}$sin2 x + C

24. Ans. B, Since, $(1 - x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots$

$\Longrightarrow \quad \frac{d}{dx}(1 - x)^{-1} = 1 + 2x + 3x^2 + 4x^3 + \ldots$

Since, derivative of $(1 - x)^{-1} = 1 + 2x + 3x^2 + 4x^3 + \ldots$

∴ Anti-derivative of $1 + 2x + 3x^2 + 4x^3 + \ldots$ is $(1 - x)^{-1}$

i.e. $\int (1 + 2x + 3x^2 + 4x^3 + \cdots \infty)$dx = $(1 - x)^{-1}$ + C

25. Ans. B, Let / = $\int (\sin x . \cos\ x . \cos 2x . \cos 4\ x . \cos 8x . \cos 16x)$dx

$= \int \sin x . \left(\frac{sin 2^5 x}{2^5 \sin x}\right)$dx

$\left[\because \cos\theta . \cos 2\theta . cos2^2\theta . cos2^4\theta \ldots cos2^{n-1}\theta = \frac{sin2^n\theta}{2^n sin\theta}\right]$

$= \frac{1}{32} \int sin32xdx = \frac{-cos32x}{32 \times 32} + C$

$= -\frac{cos32}{1024}$ + c

26. Ans. B, Let / = $\int \frac{\sin x + 4sin3x + 6 \sin 5x + 3sin7x}{sin2x + 3 \sin 4x + 3 \sin 6x}$ dx

$= \int \frac{(\sin x + \sin 3x) + 3(sin3x + sin5x) + 3(sin5x + sin7\ x)}{\sin 2\ x + 3 \sin 4x + 3sin6x}$ dx

$\left[\because \sin C + \sin D = 2sin\frac{C + D}{2} \cos\frac{C - D}{2}\right]$

$= \int \frac{2 \cos x [sin2\ x + 3 \sin 4x + 3 \sin 6x\ ]}{sin2x + 3 \sin 4x + 3 \sin 6x}$ dx

$= 2\int \cos x\ dx = 2 \sin + C$

27. Ans. B, Let / = $\int (3 \sin x\ cos^2\ x - \sin^3 x)$dx

$= \int [3 \sin x\ cos^2 x(1 - sin^2) - sin^3\ x] dx$

$= \int (3 \sin x\ - sin^3\ x - sin^3\ x) dx$

$= \int (3 \sin x - 4\ sin^3\ x) dx = \int \sin 3x dx$

$= \frac{-\cos 3x}{3} + c$

28. Ans. A, Let / = $\int \frac{(1+x)^2}{x(1+x^2)}$dx = $\int \frac{1+x^2+2x}{x(1+x^2)}$dx

$= \int \left[\frac{1+x^2}{x(1+x^2)} + \frac{2x}{(1+x^2)x}\right] dx = \int \left[\frac{1}{x} + \frac{2}{1+x^2}\right]$dx

= log x + 2 $\tan^{-1}$ x + C

29. Ans. C, Let / = $\int \frac{\cos 2x - \cos 2a}{\cos x - \cos a}$ dx

$= \int \frac{2 \cos^2 x - 1 - 2 \cos^2 a + 1}{\cos x - \cos a}$ dx

$= \int \frac{2(cos^2\ x - cos^2\ a)}{\cos x - \cos a}$dx

$= 2\int (\cos x + \cos a) dx$

= 2(sin x + xcos a) + c

30. Ans. D, Let / = $\int \frac{x^4 + x^2 + 1}{2(1+x^2)}$ dx

$= \int \frac{x^2(1+x^2)+1}{2(1+x^2)}\ dx = \int \left[\frac{x^2}{2} + \frac{1}{2(1+x^2)}\right] dx$

$=\frac{1}{2}\int\left[x^2+\frac{1}{1+x^2}\right]dx=\frac{1}{2}\left[\frac{x^3}{3}+tan^{-1}x\right]+C$